Government Disclosure in the Pandemic

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Abstract

Motivated by the pandemic COVID-19, this paper aims to explore the optimal policy for public information release during an epidemic by employing the framework of Information Design and Bayesian Persuasion. The paper formulates a model of government information disclosure to the public and, based on theoretical analysis, predicts that when the Government possesses commitment power, any partial information disclosure with a partition structure is better than no information disclosure but not as good as full information disclosure, in terms of ex-ante social welfare.

1 Introduction

Over the past decade, there has been significant progress in Information Design literature alongside Mechanism Design. In Mechanism Design, the designer influences the outcome by specifying the game that the agents will play, given the information structure. In contrast, Information Design involves the designer influencing the outcome by choosing the information structure, given the game that the agents play (Kamenica, 2019). Information Design has emerged as a promising tool for understanding and improving various problems in the modern world. Its applications range from schools disclosing grades (Ostrovsky and Schwarz, 2010; Boleslavsky and Cotton, 2015) to reducing traffic congestion through improved routing (Das et al., 2017).

Motivated by the current pandemic COVID-19, this paper seeks to explore the optimal public disclosure policy during an epidemic using the intuition from Information Design. I consider the following environment: At the early stage of an epidemic, the benevolent principal (the Government) is better informed about the disease than the Public. The Public includes a mass of individuals who are susceptible to the virus. Individuals use their prior information (private information about the likelihood of being infected/ private information channels if they could access it), learn the severity of the disease from the government disclosure, and decide how much social activities they would participate in. Private decisions of individuals pose externality to others. The Government (Sender) designs the information structure to manipulate the belief of the Public (Receivers) about how severe the disease is to maximize social welfare.

The remainder of the paper is structured as follows. The next section, Section 2 reviews briefly the literature on Information Design and Disease Transmission. Section 3 sets up the canonical model and considers both cases when the Government has commitment power and the opposite. Then, key assumptions in the basic model and their robustness are discussed in Section 4. Finally, Section 5 concludes with a summary of findings and policy implications.

2 Related Literature

2.1 Information Design

Strategic Communication Game

Sender - Receiver game model has been studied extensively in the literature. This is the situation in which an informed party (the Sender) is incentivized by the desire to sway the action of another party (the Receiver) who observes the signal realization. The Sender has full control over information disclosure about the state but cannot use any other incentive tools such as monetary transfers. Traditional games include Cheap talk (Crawford and Sobel, 1982), Verifiable message (Milgrom, 1981), and Signaling games (Spense, 1973).

Bayesian persuasion

Bayesian persuasion (Kamenica and Gentzkow, 2011) can alternatively be seen as a communication protocol. Relative to these other models of communication, Bayesian persuasion endows the Sender with more commitment power and search for the optimal information disclosure mechanism. In the most popular design, Bayesian persuasion enables the Sender to commit to any signalling rule which specifies the distribution of messages as a function of the state of the world, this rule of releasing information is common knowledge and every agent acts Bayesian-rationally.

In terms of applications, there are many situations raising the question of what is the optimal way to reveal information. For instance, a school may increase the chance that its student can acquire a job if only coarse grading policy is issued; a traffic software can reduce congestion on roads by privately sending drivers noisy information about the state of traffic; a social planner might improve everyone's welfare by providing only partial information about solvency of banks; etc.

Persuasion of an informed receiver

Persuasion of an informed receiver is the information design problem in an environment in which the receiver has access to external sources of information.

Matyskova (2018) analyzes situations where Receiver has no private information but can collect extra costly information after observing the realization of signal from Sender. She indicates that while the threat of additional information gathering can beneficial or harmful for the Receive, it weakly lessen the Sender's utility.

Moreover, Receivers might be able to obtain some exogenous information about the state but this information is not private. In this case, the Sender plays a role as an omniscient designer who knows not only the true state, but also the players' prior information about the state) can send his signal contingent upon the realizations of the exogenous signal as typical assumption in common discussions about Bayes correlated equilibria (Bergemann and Morris, 2019; Kamenica, 2019).

A popular extension is to investigate the possibility that the Receiver may possess some private information. Commonly, two following cases are distinguished from each other in literature.

Firstly, this private information might be about the state of the world. There are many examples of information disclosure problem when the Receiver has private information about the subject promoted: An entrepreneur tries to persuade an investor to fund his business project. Whilst the investor observes the result of any experiment implemented by the entrepreneur, the investor can also gather information by conducting his own assessment of the project quality and becoming privately informed (Azarmsa and Cong, 2020). A lobbyist tries to convince a policymaker about a policy issue by presenting the outcome of some related scientific studies. Considering the possibility that the lobbyist may intervene these research design through funding, the policymaker could also conducts an further inspection to obtain information about the policy problem and becomes privately informed (Minaudier, 2018).

Secondly, this private information might be about her own preferences. In the case of product information providing, Kolotilin et al. (2017) and Kolotilin (2018) assume that the receiver privately learns about his threshold for accepting and the receiver's threshold is independent of the quality. Rayo and Segal (2010) assume that the receiver has an outside option value (threshold for accepting the prospect) which is an uniformly distributed random variable independent of the prospect.

In both scenarios, the literature has also considers the Sender's ability to elicit information from Receiver prior to generating the signal. Bergemann and Morris (2019) examine the case where Receiver has private information about the state and provides the incentive conditions for a decision rule to be implementable both when the Sender can elicit the Receiver's private information and when he cannot. The findings imply that these both cases associated with the same set of implementable decision rules, nonetheless, the equivalence only holds under a binary state space with a single receiver and a binary action space. Li and Shi (2017) consider a seller (Sender) who promotes a product to a buyer (Receiver) whose private type is his own information about the quality (state). The seller performs an experiment along with an advance payment and a strike price for each type of buyer. They allow the Sender to condition his signal upon Receiver's report but does not observe the report prior to setting the price. Finally, they show that disclosing different information to different types is superiority over full disclosure. Under the same context but focusing on settings in which transfers are not possible, Guo and Shmaya (2019) consider both cases in which the sender discloses the same information to all types, and one in which the sender discloses different information to different types. They show that no privately incentive compatible (IC) mechanism could bring Sender a higher payoff than the optimal IC mechanism. Kolotilin et al. (2017) consider the model where Receiver's private information is about her own preferences, and Sender can observe Receiver's reported type. They establish the equivalent of implementation by persuasion mechanism and by experiment in linear environment, therefore, Sender could not be better off by eliciting Receiver's private information.

Persuasion of multiple receivers

Regarding to information design with multiple receivers, there are at least two players, the Receivers, and the information designer, the Sender, has an informational advantage over players. It is considered as the most important extension of Bayesian persuasion problem (with only one receiver in the basic model). There are two cases that the equivalence between the multiple-receiver environment and the single receiver one could break out. In particular, if the Sender can release separate signals to each receiver, and if there exists external effect among receivers' optimal action, the problem turns significantly more difficult. Thus, the most general case in literature includes both of these possibilities.

One common approach to Sender's optimization problem takes two steps (Bergemann and Morris, 2016a; Taneva, 2019). The first step is to identify the set of all outcomes (joint distributions over the states and receivers' actions) that can be achieved by some signal. This set of implementable outcomes corresponds to the set of Bayes-correlated equilibria (Bergemann and Morris, 2013, 2016b). In the second step, we can characterize which of these outcomes would be most profitable for the information designer by selecting the best equilibrium given some objective function. This process is analogous to the mechanism design literature: we can first determine which outcomes are implementable and then determine the one most preferred by the designer. The problem thereby is simplified to a linear program.

Another approach to the general version of information design problem with multiple receivers is proposed by Mathevet et al. (2020). According to their suggestion, we should first determine the optimal purely private signal for every prior, and then combine the optimal public signal with the optimal private signals (which are condition on the realization of the public signal) to yields the overall optimum. An advantage of this method compared to the approach via Bayes correlated equilibria is that it is still valid even if equilibrium selection may not be favorable for Sender.

Methodology for optimization problem in Information Design

Concavification, which is the standard approach to Bayesian persuasion, is firstly introduced by Kamenica and Gentzkow (2011). They propose a geometric analysis of the function mapping Receiver's posterior beliefs to Sender's indirect utility of posterior beliefs and derive the optimal mechanism by taking the concave closure of this indirect utility function. Concavifcation focuses more on the distribution of posteriors that are induced for the Receivers by a mechanism, rather than on the joint distribution between actions and states. However, the concavifcation approach provides a visual solution to the information design problem only when the state space is small, with two or three states. In addition, as Gentzkow and Kamenica (2016) criticizes, the concavification approach has some limitations when the set of Sender's types is an interval, since the set of posterior beliefs becomes infinite dimensional.

Another approach is reducing the Sender's problem of finding an optimal mechanism to a linear program, because a mechanism is described by the conditional probabilities of messages given the Sender's types, and the expected utilities are linear in these probabilities. Contrast to the concavification, this approach can be appropriate in the case where the state space is large, or even uncountable. While the concavification does not explicitly characterize the value of the problem or the optimal signal , the linear programming approach solves the dual problem: it derives necessary and sufficient conditions under which a candidate mechanism is optimal.

Imposing different assumptions, previous studies look for results on circumstances under which optimal signals fall into a certain structure class. A special case received much attention is that Sender's payoff depends only on the mean of Receiver's posterior. Ivanov (2015) and Dworczak and Martini (2019) provide necessary and sufficient conditions for optimality of monotone partition signals. Kolotilin (2018) assumes that the action space is binary and allows for Receiver to have private information about his preferences, provides necessary and sufficient conditions for optimality of a signal that reveals moderate types and hides extreme types. Guo and Shmaya (2019) show that when the action space is binary, and Receiver has private information about the state, the optimal signal has a particular structure that they term a nested interval.

Without the assumption that Sender's payoff depends only on the mean of Receiver's posterior, Mensch (2019) establishes necessary and sufficient conditions for the optimality of monotone partitions. Ivanov (2015) extends the canonical model and allows for the dependency of Sender's payoff on the rank of the realized posterior mean among the possible generated posterior means.

Rather than completely characterizing optimal signals, Kolotilin and Wolitzky (2020) develop a general approach to understanding a key qualitative property of such signals: their assortative structure, which describes the overall pattern of what states are pooled together, and how the induced receiver action varies over pairs of states. Their analysis of single-dipped and single-peaked disclosure unifies and generalizes most of the main qualitative results in the literature on persuasion with (non-linear) preferences.

With respect to Information Design Literature, my model takes into account both cases of information transmission. One is when the Government (Sender) lacks commitment power and communicates to maximize interim social welfare. This situation is first proposed by Crawford and Sobel (1982). Another is when Government (Sender) has commitment power and commits to a signaling rule to maximize its ex-ante expected utility, as introduced by (Kamenica and Gentzkow, 2011). Different from most of the previous papers, my paper analyzes the model of public information disclosure to multiple receivers with the presence of externality, without restricting that Sender's payoff depends only on the mean of Receiver's posterior.

2.2 Disease Transmission

Modeling the process of disease transmission has drawn attention for a century with the original susceptible-infected-recovered (SIR) model of Kermack and McKendrick (1927) and has become the focus of a great influx of recent economics papers in the context of the Covid-19 pandemic.

Based on the seminal work of Kermack and McKendrick (1927), later extensions attempt to integrate the individually optimized choices (Geoffard and Philipson, 1996; Chen, 2012; Farboodi et al., 2020). Chen (2012) argues that the decision of individuals on choosing their levels of public avoidance during an epidemic entails an evaluation of the relative cost and benefit of staying home and minimizing interaction with others. Moreover, such individual evaluation also depend considerably on the action of other people. This study analyzes both the set of Nash equilibria of the model and makes comparison to the social optimum.

Among the latest developments in this literature, some studies take into consideration the role of information and communication on disease transmission given the uncertainty about the disease' severity.

Bursztyn et al. (2020) investigate the extent to which misinformation broadcast on mass media at the early stages of the COVID-19 pandemic affects individual behavior and health outcomes. Given the large externalities inherent to contagious diseases, they find that areas with greater exposure to the show downplaying the threat of COVID-19 experienced a greater number of cases and deaths.

Related to the politicization of COVID-19, many studies (Mariani et al., 2020; Rafkin et al., 2020; Allcott et al., 2020; Kushner Gadarian et al., 2020; Painter and Qiu, 2020) provide empirical evidences that how policymakers and public servants communicate about the severity of the epidemic and their recommendations to the public could influence individual compliance, thereby the efficiency of social distancing policy, especially in highly polarized contexts. Mariani et al. (2020) show that president Bolsonaro's display of skepticism substantially caused a divergence in COVID-19 diffusion trends between these two groups of municipalities in Brazil, where, faster spread of the virus in municipalities that concentrate his supporters. Using an online experiment US respondents, Rafkin et al. (2020) indicate that inconsistent positions from the federal government reduce people's belief updates when presented with official information.

From theoretical perspective, consider the effect of information disclosure by government, Yue and Yixi (2020) accommodate the information asymmetry in the model and analyze the equilibrium strategic communication during an epidemic. Their results predict that the incentive misalignment between the government (Sender) and the public (Receiver) tend to prohibit truthful communication. Such communication failure is the consequence of an assumption that government lacks of pre-commitment power. It sounds reasonable in many countries since the regulator could hardly commit truth-telling in an unprecedented shock as the COVID-19 pandemic.

While employing a variant game theory model with strategic interaction between action of individuals in an epidemic as proposed by Chen (2012), my approach allows the incomplete information among the public and consider the role of government in disclosing information about state of the world. I relax the assumption of Government's commitment power in Yue and Yixi (2020) to examine whether Government could be able to further improve social welfare using persuasion strategies as the Information Designer and what is the optimal signaling rule. To the best of my knowledge, this paper is the first attempt to apply Bayesian framework to Government disclosure problem in an epidemic.

3 Canonical Model

3.1 Setup

Individuals. There is a unit mass of homogeneous utility maximizing individuals. A representative individual chooses an level of social activity $a \in [0, 1]$ with the least level is staying home as well as forgoing certain activities, for instance, going to school/work, shopping or joining social events. An individual's payoff from the social activity is $v(a)$ which is an increasing and concave function on $[0, 1]$ (i.e. satisfies assumption of diminishing marginal utility). Moreover, there is a risk of getting infected associated with taking social activities, which induces an extra dis-utility amount.

Infection process. Assume that in the initial stage of the epidemic, there is a small fraction $i \in (0, 1)$ of the population are infected, and in their incubation period. Also assume that all people who are infected with symptoms are quarantined and have no social activity choices to make, while people in the incubation period almost display no symptoms, so they are uncertain whether they are infected or not, and make social activity decisions as if they were not infected. However, with the pass-through rate $\beta \in (0, 1]$, infected people in the incubation period could infect the others they meet. If the individual belongs to $1 - i$ of the population who have not been infected, then when he/she chooses social activity level a, the probability that he/she could be infected is $a\lambda$, where λ denotes the probability of being infected per unit social activity level at the same time period. This probability is a function of the disease prevalence i, pass-through rate β and the average social activity chosen by all individuals. If all other individual choose some level z, λ can be specified as follows:

$$\lambda = \beta i z$$

Assume that the loss from infection is θ, an individual who chooses level of social activity a obtains the expected utility:

$$U_P = v(a) - (1-i)\beta i z a \theta - i\theta$$

since the expected cost is a weighted sum of the expected loss from having not been infected but getting infected through social activities and the loss if already infected, with the weights are probabilities the individual belongs to each group respectively.
Hence,

$$U_P \approx v(a) - \beta i z a \theta - i\theta$$

(as i is very small)

Government. As a benevolent social planner, government wants to maximize social welfare (the sum of the utility of all individual in population). Because individuals are homogeneous by above assumption, the number of individuals does not matter, thus social planner's problem is simplified to:

$$Max_{a\in[0,1]} U_G = v(a) - \beta i a^2 \theta - i\theta$$

Information Transmission. Assume that government is better informed about the severity of disease (e.g. the death rate, the pressure on health system) than the public. The public is uncertain about the loss from infection, Θ, and share an i.i.d common prior belief μ_0 on support $[\underline{\theta}, \overline{\theta}]$. However, government knows exactly the realization θ of Θ. Other aspects of the disease (pass-through rate β, fraction of infected i) are common knowledge to everyone.

Timing of the game is as follows: First, the government (Sender) observes the loss of being infected, θ, and sends a signal $s \in S$ to the public (Receivers). Second, the public receives the government's signal, updates its belief on Θ, and optimally chooses the social activity level a, which affects both Sender and Receiver's utility.

For the sake of comparison, I consider two cases of information transmission game:

(1) The Government (Sender) lacks commitment power, and communicates to maximize its interim utility (Cheap talk - Crawford and Sobel (1982)).

(2) The Government (Sender) has commitment power, and choose how to disclosure information to maximize its ex-ante expected utility (Bayesian persuasion - Kamenica and Gentzkow (2011)).

CASE 1. The Government (Sender) lacks pre-commitment power.
In this case, Sender's choice of signalling rule and Receiver's choice of action rule are strategically "simultaneous". With $s \in S$ is the signal space, define:

- Government's strategy is an signalling rule which is a distribution $\sigma(s|\theta) \in [0,1]$ for each state θ of the world.

- Public's strategy is an action rule $\rho(s) : S \mapsto [0,1]$, which determines action taken upon receipt of signal s.

- Public's posterior belief on Θ is $\mu(\theta|s) : [\underline{\theta}, \overline{\theta}] \mapsto [0,1]$, which specifies the belief formed upon receipt of signal s using Bayes' Rule.

The solution is weak Perfect Bayesian equilibrium $\{\sigma(s|\theta), \rho(s), \mu(\theta|s)\}$, which satisfies:

- For each θ, $\int_S \sigma(s|\theta)ds = 1$, if s^* is in the support of $\sigma(.|\theta)$, then s^* solves: $\max_S U_G(\rho(s), \theta)$.

- For each s, $\rho(s)$ solves: $\max_a \int_S U_P(a,\theta)\mu(\theta|s)ds$, where $\mu(\theta|s) = \frac{\sigma(s|\theta)\mu_0(\theta)}{\int_\Theta \sigma(s|\theta')d\mu_0(\theta')}$.

CASE 2. The Government (Sender) has commitment power.
In this case, Sender plays a role as information designer, who commits to a signalling rule in advance. This rule becomes common knowledge of both Sender and Receiver.

- Given their knowledge of σ, Public use Bayes' Rule to update their belief from the prior μ_0 to the posterior $\mu(\theta|s) = \frac{\sigma(s|\theta)\mu_0(\theta)}{\int_\Theta \sigma(s|\theta')d\mu_0(\theta')}$, and then they select action $\rho(s)$ which solves $\max_a \int_S U_P(a,\theta)\mu(\theta|s)ds$.

- Given this behavior of Public, Government choose optimal signal rule σ that solves:

$$\max_\sigma \int_{\underline{\theta}}^{\overline{\theta}} \int_S U_G(\rho(s),\theta)d\sigma(s|\theta)d\mu_0(\theta)$$

Partition Structure. We will focus on the signalling rule that has partition structure with two key properties:

- State space is divided into k subintervals denoted $[\theta_{j-1}, \theta_j]$, where $\theta_0 = \underline{\theta}$ and $\theta_k = \overline{\theta}$

- Signal sent depends only on the subinterval. Specifically, Sender sends only $s_1 < s_2 < ... < s_k$ such that: $\sigma(s_j|\theta) = 1 \ \forall\theta \in [\theta_{j-1}, \theta_j]$ (i.e. $\sigma(s_j|[\theta_{j-1}, \theta_j]) = 1$).

To characterize how information transmitted, we also define three types of information disclosure: No information disclosure; Partial information disclosure; Full information disclosure.

Prior to further analysis in two cases of information transmission, for simplicity, we impose additional assumption as follows:

- $v(a) = a - \frac{a^2}{4}$
- $\mu_0(\theta)$ is uniform. As a result, $\mu(\theta|s_j) = U_{[\theta_{j-1},\theta_j]} \ \forall j = \overline{1, k}$

Then, the respective utility functions for each individual and the Government are:

$$U_P = a - \frac{a^2}{4} - \beta i z a \theta - i\theta$$
$$U_G = a - \frac{a^2}{4} - \beta i a^2 \theta - i\theta$$

Consider full information as benchmark (when the Government has no information advantage compared to the Public). In this circumstance, there is an unique symmetric Nash equilibrium of the Public is that everyone choose level $a = \frac{2}{1+2\beta i\theta}$, which is higher than the social optimum: $\frac{2}{1+4\beta i\theta}$ for all the state θ. Therefore, Government and Public's objectives are not aligned. Indeed:

- Given action z of others, knowing the state θ each individual will select action $\rho(\theta)$ to maximize:
  $$U_P = a - \frac{a^2}{4} - \beta i z a \theta - i\theta$$
  F.O.C.
  $$1 - \frac{a}{2} - \beta i z \theta = 0$$
  $\Rightarrow \rho(\theta) = 2 - \beta i z \theta$

  Since all individuals have the same preference structure by assumption, we focus on symmetric equilibria in which all individuals choose the same social activity level $z = \rho(\theta)$. Hence:
  $$\rho(\theta) = 2 - 2\beta i \rho(\theta)\theta$$
  $$\rho(\theta) = \frac{2}{1 + 2\beta i\theta}$$

- Government wants to select action a to maximize:

$$U_G = a - \frac{a^2}{4} - \beta i a^2 \theta - i\theta$$

F.O.C.

$$1 - \frac{a}{2} - 2\beta i \theta a = 0$$
$$\Rightarrow a = \frac{2}{1 + 4\beta i \theta}$$

- As it can be seen, individuals' social activity levels are strategic substitutes, and there is a unique Nash equilibrium. The social optimum which the Government desires to attain is also unique. Nonetheless, at the social optimum, each individual who behaves to maximize his/her own utility would be better off increasing his/her level of activity. As a result, level of social interactions during the epidemic exceeds what is the best for population as a whole.

3.2 Government has no ex-ante commitment power

Proposition 1.

In the case that Government has no ex-ante commitment power, we have no information disclosure in the equilibrium (babbling equilibrium is the unique equilibrium): Government always sends the same signal in every state, while Public always hold their prior beliefs and choose the social activity level equal to $\frac{2}{1+2\beta i \mathbb{E}_{\mu_0}[\theta]}$.
Public under reaction happens whenever $\theta > \mathbb{E}_{\mu_0}[\theta]$, whereas Public over reaction happens whenever $\theta < \mathbb{E}_{\mu_0}[\theta]$. Both optimal solution for individuals and for social welfare are not achieved in these cases.

Proof.
From the Government's perspective, we will show that no information disclosure (sending the same signal in every state) is always best response given any strategy (action rule) of the Public. In other words, the optimal signalling rule is $\sigma(s_1|\theta) = 1 \ \forall \theta \in [\underline{\theta}, \overline{\theta}]$.

By contradiction, suppose that the equilibrium signalling rule σ^* is different from no information disclosure. Clearly, there exists at least two different $s_j, s_{j'}$ where the posterior beliefs are not the same, i.e. $\mu(\theta|s_j) \neq \mu(\theta|s_{j'})$ for some θ.[1]

[1] Otherwise, for any two signals $s_j, s_{j'}$ and any $\theta \in [\underline{\theta}, \overline{\theta}]$, $\mu(\theta|s_j) = \mu(\theta|s_{j'}) \Rightarrow \mu(\theta|s) = \mu_0(\theta)$ for any s, θ (by Bayes Rule). Thus, all messages are uninformative.

Notice that in equilibrium $\rho(s) = \arg\max \int_S U_P(a,\theta)\mu(\theta|s)ds$, therefore:

- If $\rho(s_j) < \rho(s_{j'})$, then $U_G(\rho(s_j),\theta) > U_G(\rho(s'_j),\theta)$, so $s_{j'}$ can not be induced in equilibrium, a contradiction. Similar for $\rho(s_j) > \rho(s_{j'})$.

- If $\rho(s_j) = \rho(s_{j'})$, we can obtain another equivalent equilibrium by replace s_j and $s_{j'}$ by other signal $\hat{s}$. Keep doing this process until there is no signal pair that induces different posterior beliefs, we obtain the signalling rule $\hat{\sigma}$ is equivalent with the initial signalling rule σ^*. Then, σ^* is no information disclosure, a contradiction.

Therefore, in equilibrium, the Government always sends the same signal in every state. Given that signalling rule, the Public always hold their prior beliefs and select the social activity level equal to $\frac{2}{1+2\beta i\mathbb{E}_{\mu_0}[\theta]}$.

Comparing to the full information benchmark, we obtain the remains of Proposition 1 intermediately.

3.3 Government has ex-ante commitment power

Proposition 2.

In the case that Government has ex-ante commitment power, full information disclosure is optimal. Furthermore, denote EU_{Gfull}, $EU_{Gpartial}$ and EU_{Gno} are expected social welfare when Government choose full information disclosure, partial information disclosure and no information disclosure respectively.
Then: $EU_{Gfull} > EU_{Gpartial} > EU_{Gno}$.

Proof.
Firstly, consider all signalling rule $\sigma(s|\theta)$ with partition structure that has k partitions ($k \in \mathbb{N}^*$). Upon receiving signal s_j ($j = \overline{1,k}$), each individual know that θ is uniformly distributed in $[\theta_{j-1}, \theta_j]$. Given action z of others, he/she will select action $\rho(s_j)$ to maximize:

$$\int\limits_{\theta_{j-1}}^{\theta_j} (a - \frac{a^2}{4} - \beta iza\theta - i\theta)\frac{1}{\theta_j - \theta_{j-1}}d\theta$$

F.O.C.

$$\int\limits_{\theta_{j-1}}^{\theta_j} (1 - \frac{a}{2} - \beta iz\theta)\frac{1}{\theta_j - \theta_{j-1}}d\theta = 0$$

$$\left[(1 - \frac{a}{2})\theta - \beta iz\frac{\theta^2}{2}\right]_{\theta_{i-1}}^{\theta_i} = 0$$

$\Rightarrow \rho(s_j) = 2 - \beta iz(\theta_i + \theta_{i-1})$
Since all individuals have the same preference structure by assumption, we focus on symmetric equilibria in which all individuals choose the same social activity level $z = \rho(s_j)$. Hence,

$$\rho(s_j) = 2 - \beta i\rho(s_j)(\theta_j + \theta_{j-1})$$
$$\rho(s_j) = \frac{2}{1 + \beta i(\theta_j + \theta_{j-1})}$$

Hence,

$$\int_{\theta_{j-1}}^{\theta_j} U_G(\rho(s_j),\theta)d\theta$$
$$= \int_{\theta_{j-1}}^{\theta_j} \left[\frac{2}{1+\beta i(\theta_j+\theta_{j-1})} - (\frac{1}{4}+\beta i\theta)(\frac{2}{1+\beta i(\theta_j+\theta_{j-1})})^2 - i\theta\right] d\theta$$
$$= \left\{\frac{2}{1+\beta i(\theta_j+\theta_{j-1})} - \frac{1}{[1+\beta i(\theta_j+\theta_{j-1})]^2}\right\}\theta\ |_{\theta_{j-1}}^{\theta_j} - \left\{\frac{4\beta i}{[1+\beta i(\theta_j+\theta_{j-1})]^2}+i\right\}\frac{\theta^2}{2}\ |_{\theta_{j-1}}^{\theta_j}$$
$$= \frac{\theta_j-\theta_{j-1}}{[1+\beta i(\theta_j+\theta_{j-1})]^2} - \frac{(\theta_j^2-\theta_{j-1}^2)i}{2}$$

Therefore,

$$EU_{Gpartial} = \frac{1}{\overline{\theta}-\underline{\theta}}\sum_{j=1}^{k}\int_{\theta_{j-1}}^{\theta_j} U_G(\rho(s_j),\theta)d\theta$$
$$EU_{Gpartial} = \frac{1}{\overline{\theta}-\underline{\theta}}\sum_{j=1}^{k}\left\{\frac{\theta_j-\theta_{j-1}}{[1+\beta i(\theta_j+\theta_{j-1})]^2} - \frac{(\theta_j^2-\theta_{j-1}^2)i}{2}\right\}$$
$$EU_{Gpartial} = \frac{1}{\overline{\theta}-\underline{\theta}}\sum_{j=1}^{k}\frac{\theta_j-\theta_{j-1}}{[1+\beta i(\theta_j+\theta_{j-1})]^2} - \frac{(\overline{\theta}+\underline{\theta})i}{2}$$

Similarly, we obtain results for special cases:

No information disclosure. This is equivalent to the case $k=1$, thus:

$$EU_{Gno} = \frac{1}{\left[1+\beta i(\overline{\theta}+\underline{\theta})\right]^2} - \frac{(\overline{\theta}+\underline{\theta})i}{2}$$

Full information disclosure.

$$EU_{Gfull} = \int_{\underline{\theta}}^{\overline{\theta}} U_G(\rho(\theta),\theta)\frac{1}{\overline{\theta}-\underline{\theta}}d\theta$$
$$EU_{Gfull} = \frac{1}{\overline{\theta}-\underline{\theta}}\int_{\underline{\theta}}^{\overline{\theta}}\left[\frac{2}{1+2\beta i\theta} - (\frac{1}{4}+\beta i\theta)(\frac{2}{1+2\beta i\theta})^2 - i\theta\right]d\theta$$
$$EU_{Gfull} = \frac{1}{(1+2\beta i\overline{\theta})(1+2\beta i\underline{\theta})} - \frac{(\overline{\theta}+\underline{\theta})i}{2}$$

Since $[1+\beta i(\theta_j+\theta_{j-1})]^2 > (1+2\beta i\theta_j)(1+2\beta i\theta_{j-1})$, $EU_{Gpartial} < EU_{Gfull}$.
Since $\sum_{j=1}^{k}(\theta_j-\theta_{j-1})\sum_{j=1}^{k}\frac{\theta_j-\theta_{j-1}}{[1+\beta i(\theta_j+\theta_{j-1})]^2} \geq (\sum_{j=1}^{k}\frac{\theta_j-\theta_{j-1}}{1+\beta i(\theta_j+\theta_{j-1})})^2 \geq (\frac{\overline{\theta}-\underline{\theta}}{1+\beta i(\overline{\theta}+\underline{\theta})})^2$, $EU_{Gpartial} > EU_{Gno}$.
In summary, $EU_{Gfull} > EU_{Gpartial} > EU_{Gno}$ (Q.E.D)

4 Extensions

In this section, we will focus on the case 2,[2] when Government has commitment power as Bayesian Persuasion framework, to discuss assumptions in the canonical model and check their robustness.

4.1 Payoff function $v(a)$

Consider the case that function $v(a)$ is strictly increasing (v'>0) and strictly concave (v”<0), and satisfies Inada conditions (i.e. $\lim_{a \to \infty} v'(a) = 0; \lim_{a \to 0} v'(a) = \infty$).

Full information disclosure. Given state θ, there is a unique symmetric equilibrium at which everyone chooses action $\rho(\theta)$ satisfying:

$$v'(\rho(\theta)) = \beta i \theta \rho(\theta) \tag{1}$$

Since the right-hand side is linear and increasing function of $\rho(\theta)$, with above assumption of payoff function $v(.)$, equation (1) has a unique solution $\rho(\theta)$ for each θ.

Partial information disclosure. Consider all signalling rule $\sigma(s|\theta)$ with partition structure that has k partitions ($k \in \mathbb{N}^*$). Upon receiving signal s_j ($j = \overline{1,k}$), each individual know that θ is uniformly distributed in $[\theta_{j-1}, \theta_j]$. Thus, there is a unique symmetric equilibrium at which everyone chooses action $\rho(s_j)$ satisfying:

$$v'(\rho(s_j)) = \beta i \frac{\theta_j + \theta_{j-1}}{2} \rho(s_j)$$
$$\Rightarrow \rho(s_j) = \rho(\frac{\theta_j + \theta_{j-1}}{2}) \; \forall j = \overline{1,k}$$

No information disclosure. This is equivalent to the case $k = 1$. Upon receiving the same signal s, each individual has no additional information. Thus, there is a unique symmetric equilibrium at which everyone chooses action $\rho(s)$ satisfying:

$$v'(\rho(s)) = \beta i \frac{\overline{\theta} + \underline{\theta}}{2} \rho(s)$$
$$\Rightarrow \rho(s) = \rho(\frac{\overline{\theta} + \underline{\theta}}{2})$$

[2]The result for case 1 is unaffected by assumptions which we consider here. It can be established in a very similar way to Proposition 1.

Therefore:

$$EU_{Gfull} = \int_{\underline{\theta}}^{\overline{\theta}} U_G(\rho(\theta), \theta) \frac{1}{\overline{\theta} - \underline{\theta}} d\theta$$

$$= \frac{1}{\overline{\theta} - \underline{\theta}} \int_{\underline{\theta}}^{\overline{\theta}} \left[v(\rho(\theta)) - \beta i \theta (\rho(\theta))^2 \right] d\theta - \frac{(\overline{\theta} + \underline{\theta}) i}{2}$$

$$EU_{Gpartial} = \frac{1}{\overline{\theta} - \underline{\theta}} \sum_{j=1}^{k} \int_{\theta_{j-1}}^{\theta_j} U_G(\rho(\theta), \theta) d\theta$$

$$= \frac{1}{\overline{\theta} - \underline{\theta}} \sum_{j=1}^{k} \left\{ v\left(\rho\left(\frac{\theta_j + \theta_{j-1}}{2}\right)\right)(\theta_j - \theta_{j-1}) - \left[\beta i \left(\rho\left(\frac{\theta_j + \theta_{j-1}}{2}\right)\right)^2 + i\right] \frac{\theta_j^2 - \theta_{j-1}^2}{2} \right\}$$

$$= \sum_{j=1}^{k} \frac{\theta_j - \theta_{j-1}}{\overline{\theta} - \underline{\theta}} v\left(\rho\left(\frac{\theta_j + \theta_{j-1}}{2}\right)\right) - \sum_{j=1}^{k} \frac{\theta_j^2 - \theta_{j-1}^2}{2(\overline{\theta} - \underline{\theta})} \beta i \left(\rho\left(\frac{\theta_j + \theta_{j-1}}{2}\right)\right)^2 - \frac{(\overline{\theta} + \underline{\theta}) i}{2}$$

$$EU_{Gno} = v\left(\rho\left(\frac{\overline{\theta} + \underline{\theta}}{2}\right)\right) - \frac{\overline{\theta} + \underline{\theta}}{2} \beta i \left(\rho\left(\frac{\overline{\theta} + \underline{\theta}}{2}\right)\right)^2 - \frac{(\overline{\theta} + \underline{\theta}) i}{2}$$

For the sake of comparison, we employs the following property of the concave function $v(.)$: $v(a) - v(a_0) \leq v'(a_0)(a - a_0)$ (i.e. $v(a_0) - v(a) \geq v'(a_0)(a_0 - a)$) and equation (1). Then:

(I) $EU_{Gfull} - EU_{Gpartial}$

$$= \sum_{j=1}^{k} \frac{1}{\overline{\theta} - \underline{\theta}} \int_{\theta_{j-1}}^{\theta_j} \left[v(\rho(\theta)) - v\left(\rho\left(\frac{\theta_j + \theta_{j-1}}{2}\right)\right) \right] d\theta$$

$$- \sum_{j=1}^{k} \frac{1}{\overline{\theta} - \underline{\theta}} \int_{\underline{\theta}}^{\overline{\theta}} \left[\beta i \theta (\rho(\theta))^2 - \beta i \theta \left(\rho\left(\frac{\theta_j + \theta_{j-1}}{2}\right)\right)^2 \right] d\theta$$

$$\geq \sum_{j=1}^{k} \frac{1}{\overline{\theta} - \underline{\theta}} \int_{\theta_{j-1}}^{\theta_j} v'(\rho(\theta)) \left[\rho(\theta) - \rho\left(\frac{\theta_j + \theta_{j-1}}{2}\right) \right] d\theta$$

$$- \sum_{j=1}^{k} \frac{1}{\overline{\theta} - \underline{\theta}} \int_{\underline{\theta}}^{\overline{\theta}} \beta i \theta \left[(\rho(\theta))^2 - \left(\rho\left(\frac{\theta_j + \theta_{j-1}}{2}\right)\right)^2 \right] d\theta$$

$$= \sum_{j=1}^{k} \frac{1}{\overline{\theta} - \underline{\theta}} \int_{\theta_{j-1}}^{\theta_j} \beta i \theta \rho(\theta) \left[\rho(\theta) - \rho\left(\frac{\theta_j + \theta_{j-1}}{2}\right) \right] d\theta$$

$$- \sum_{j=1}^{k} \frac{1}{\overline{\theta} - \underline{\theta}} \int_{\underline{\theta}}^{\overline{\theta}} \beta i \theta \left[(\rho(\theta))^2 - \left(\rho\left(\frac{\theta_j + \theta_{j-1}}{2}\right)\right)^2 \right] d\theta$$

$$= \sum_{j=1}^{k} \frac{1}{\overline{\theta} - \underline{\theta}} \int_{\theta_{j-1}}^{\theta_j} \beta i \theta \rho \left(\frac{\theta_j + \theta_{j-1}}{2} \right) \left[\rho \left(\frac{\theta_j + \theta_{j-1}}{2} \right) - \rho(\theta) \right] d\theta$$

$$= \sum_{j=1}^{k} \rho \left(\frac{\theta_j + \theta_{j-1}}{2} \right) \frac{1}{\overline{\theta} - \underline{\theta}} \left[\frac{\theta_j^2 - \theta_{j-1}^2}{2} \beta i \rho \left(\frac{\theta_j + \theta_{j-1}}{2} \right) - \int_{\theta_{j-1}}^{\theta_j} \beta i \theta \rho(\theta) d\theta \right]$$

$$= \sum_{j=1}^{k} \rho \left(\frac{\theta_j + \theta_{j-1}}{2} \right) \frac{\theta_j - \theta_{j-1}}{\overline{\theta} - \underline{\theta}} \left[v' \left(\rho \left(\frac{\theta_j + \theta_{j-1}}{2} \right) \right) - \int_{\theta_{j-1}}^{\theta_j} v'(\rho(\theta)) \cdot \frac{1}{\theta_j - \theta_{j-1}} d\theta \right]$$

$$= \sum_{j=1}^{k} \rho \left(\frac{\theta_j + \theta_{j-1}}{2} \right) \frac{\theta_j - \theta_{j-1}}{\overline{\theta} - \underline{\theta}} \left[v' \left(\rho \left(\frac{\theta_j + \theta_{j-1}}{2} \right) \right) - \frac{v'(\rho(\theta_j)) + v'(\rho(\theta_{j-1}))}{2} \right]$$

Here, if $v'(\rho(\theta)) = \beta i \theta \rho(\theta)$ is a strictly concave function of θ, we have:

$$v' \left(\rho \left(\frac{\theta_j + \theta_{j-1}}{2} \right) \right) > \frac{v'(\rho(\theta_j)) + v'(\rho(\theta_{j-1}))}{2}$$

$$\Rightarrow \quad EU_{Gfull} - EU_{Gpartial} > 0 \quad \Rightarrow \quad EU_{Gfull} > EU_{Gpartial}$$

NOTE (∗):

So far, our task has become determining which assumptions of payoff function v we need to impose further in order that $v'(\rho(\theta)) = \beta i \theta \rho(\theta)$ is a strictly concave function of θ. As $v'(\rho(\theta)) = \beta i \theta \rho(\theta)$, we consider the function f satisfying:

$$f(\theta, \rho(\theta)) = \frac{v'(\rho(\theta))}{\rho(\theta)} - \beta i \theta = 0$$

As $\frac{\partial f(\theta,\rho(\theta))}{\partial \theta} = -\beta i$ and $\frac{\partial f(\theta,\rho(\theta))}{\partial \rho} = \frac{\rho v'' - v'}{\rho^2}$, by Implicit Theorem:

$$\rho_\theta(\theta) = -\frac{\frac{\partial f(\theta,\rho(\theta))}{\partial \rho}}{\frac{\partial f(\theta,\rho(\theta))}{\partial \theta}} = \frac{\beta i \rho^2}{\rho v'' - v'} < 0$$

Therefore,

$$\frac{dv'(\rho(\theta))}{d\theta} = \frac{dv'}{d\rho} \frac{d\rho}{d\theta} = \frac{\beta i \rho^2 v''}{\rho v'' - v'}$$

$$\frac{d^2 v'(\rho(\theta))}{d\theta^2} = \left[\frac{d}{d\rho} \left(\frac{\beta i \rho^2 v''}{\rho v'' - v'} \right) \right] \frac{d\rho}{d\theta} = \beta i \rho_\theta(\theta) \frac{d}{d\rho} \left(\frac{\rho^2 v''}{\rho v'' - v'} \right)$$

Note that

$$\frac{d}{d\rho} \left(\frac{\rho^2 v''}{\rho v'' - v'} \right) = \frac{2\rho v'(\rho v'' - v') - \rho^2 v' v'''}{(\rho v'' - v')^2} = \frac{\rho}{(\rho v'' - v')^2} \left[2v'(\rho v'' - v') - \rho v' v''' \right]$$

$$= \frac{\rho}{(\rho v'' - v')^2} \frac{-\rho^3 (v'')^2}{v'} \frac{d}{d\rho} \left[\left(\frac{v'}{\rho} \right)^2 \frac{1}{-v''} \right] = \frac{-\rho^4 (v'')^2}{v'(\rho v'' - v')^2} \frac{d}{d\rho} \left[\left(\frac{v'}{\rho} \right)^2 \frac{1}{-v''} \right]$$

Since $v' > 0, v'' < 0, \rho > 0$ and $\rho_\theta(\theta) < 0$, we obtain that $v'(\rho(\theta)) = \beta i\theta\rho(\theta)$ is a strictly concave function of θ is equivalent to:

$$\frac{d^2 v'(\rho(\theta))}{d\theta^2} < 0 \Leftrightarrow \frac{d}{d\rho}\left(\frac{\rho^2 v''}{\rho v'' - v'}\right) > 0 \Leftrightarrow \frac{d}{d\rho}\left[\left(\frac{v'}{\rho}\right)^2 \frac{1}{-v''}\right] < 0 \Leftrightarrow \frac{d}{d\rho}\left(\frac{v'}{\rho} \div \frac{-\rho v''}{v'}\right) < 0 \tag{2}$$

Now, we can characterize the sufficient condition for payoff function v such that condition (2) is satisfied. In particular, for all utility function which is CRRA (Constant Relative Risk Aversion) or IRRA (Increasing Relative Risk Aversion), with the coefficient $R(a) = -\frac{av''(a)}{v'(a)}$ is constant or increasing respectively, condition (2) will be true. This is due to that $\frac{v'}{\rho}$ is always increasing in ρ, given the assumption that $v'' < 0$. Our previous example in section 3 with $v(a) = a - \frac{a^2}{4}$ or a more general utility function in quadratic form is IRRA, so (2) and thereby $EU_{Gfull} > EU_{Gpartial}$ holds in this case. Furthermore, CRRA utility function (such that $v(a) = \frac{a^{1-\eta}-1}{1-\eta}$ or the special case is $v(a) = log(a)$ when $\eta \to 1$) is very popular in economics theory. However, (2) may be true or not true with some DRRA (Decreasing Relative Risk Aversion) utility function. Let's consider the example: $v(a) = \frac{(a-\gamma)^{1-\eta}-1}{1-\eta}$, where $\gamma > 0$ and $\eta > 0$ are given. Then, v is strictly increasing concave function and DRRA for $a > \gamma$, and:

$$\frac{d}{d\rho}\left(\frac{v'}{\rho} \div \frac{-\rho v''}{v'}\right) = \frac{(a-\gamma)^{-\eta}}{a^3\eta}\left[2\gamma - a(1+\eta)\right]$$

Hence, (2) is true for the class of functions v associated with $\eta > 1$ since, if so, $2\gamma < 2a < a(1+\eta)\ \forall a$. By contrast, (2) is not true for the class of functions v corresponding to $\eta < 1$ because in this case, $2\gamma > a(1+\eta)$ for $a \in (\gamma, \frac{2\gamma}{1+\eta})$. Therefore, we cannot assert surely that when payoff function v is IRRA, full information disclosure could achieve higher ex-ante social welfare compared to partial information disclosure from Government's perspective.

(II) $EU_{Gpartial} - EU_{Gno}$

$$= \sum_{j=1}^{k} \frac{\theta_j - \theta_{j-1}}{\overline{\theta} - \underline{\theta}}\left[v\left(\rho\left(\frac{\theta_j + \theta_{j-1}}{2}\right)\right) - v\left(\rho\left(\frac{\overline{\theta} + \underline{\theta}}{2}\right)\right)\right]$$
$$- \sum_{j=1}^{k} \frac{\theta_j^2 - \theta_{j-1}^2}{2(\overline{\theta} - \underline{\theta})}\left[\beta i\left(\rho\left(\frac{\theta_j + \theta_{j-1}}{2}\right)\right)^2 - \beta i\left(\rho\left(\frac{\overline{\theta} + \underline{\theta}}{2}\right)\right)^2\right]$$

$$\geq \sum_{j=1}^{k} \frac{\theta_j - \theta_{j-1}}{\overline{\theta} - \underline{\theta}} v'\left(\rho\left(\frac{\theta_j + \theta_{j-1}}{2}\right)\right)\left[\rho\left(\frac{\theta_j + \theta_{j-1}}{2}\right) - \rho\left(\frac{\overline{\theta} + \underline{\theta}}{2}\right)\right]$$
$$- \sum_{j=1}^{k} \frac{\theta_j^2 - \theta_{j-1}^2}{2(\overline{\theta} - \underline{\theta})}\beta i\left[\left(\rho\left(\frac{\theta_j + \theta_{j-1}}{2}\right)\right)^2 - (\rho(\frac{\overline{\theta} + \underline{\theta}}{2}))^2\right]$$

$$= \sum_{j=1}^{k} \frac{\theta_j^2 - \theta_{j-1}^2}{2(\overline{\theta} - \underline{\theta})} \beta i \rho \left(\frac{\theta_j + \theta_{j-1}}{2} \right) \left[\rho \left(\frac{\theta_j + \theta_{j-1}}{2} \right) - \rho \left(\frac{\overline{\theta} + \underline{\theta}}{2} \right) \right]$$
$$- \sum_{j=1}^{k} \frac{\theta_j^2 - \theta_{j-1}^2}{2(\overline{\theta} - \underline{\theta})} \beta i \left[(\rho(\frac{\theta_j + \theta_{j-1}}{2}))^2 - (\rho(\frac{\overline{\theta} + \underline{\theta}}{2}))^2 \right]$$

$$= \sum_{j=1}^{k} \frac{\theta_j^2 - \theta_{j-1}^2}{2(\overline{\theta} - \underline{\theta})} \beta i \left[\rho \left(\frac{\overline{\theta} + \underline{\theta}}{2} \right) - \rho \left(\frac{\theta_j + \theta_{j-1}}{2} \right) \right] \rho \left(\frac{\overline{\theta} + \underline{\theta}}{2} \right)$$
$$= \left[\frac{\overline{\theta} + \underline{\theta}}{2} \rho \left(\frac{\overline{\theta} + \underline{\theta}}{2} \right) - \sum_{j=1}^{k} \frac{\theta_j - \theta_{j-1}}{\overline{\theta} - \underline{\theta}} . \frac{\theta_j + \theta_{j-1}}{2} \rho \left(\frac{\theta_j + \theta_{j-1}}{2} \right) \right] \beta i \rho \left(\frac{\overline{\theta} + \underline{\theta}}{2} \right)$$
$$= \left[v' \left(\rho \left(\frac{\overline{\theta} + \underline{\theta}}{2} \right) \right) - \sum_{j=1}^{k} \frac{\theta_j - \theta_{j-1}}{\overline{\theta} - \underline{\theta}} v' \left(\rho \left(\frac{\theta_j + \theta_{j-1}}{2} \right) \right) \right] \rho \left(\frac{\overline{\theta} + \underline{\theta}}{2} \right)$$

Here, if $v'(\rho(\theta)) = \beta i \theta \rho(\theta)$ is a strictly concave function of θ as obtained by **NOTE (∗)**, we also have:

$$\sum_{j=1}^{k} \frac{\theta_j - \theta_{j-1}}{\overline{\theta} - \underline{\theta}} v' \left(\rho \left(\frac{\theta_j + \theta_{j-1}}{2} \right) \right) < v' \left(\rho \left(\sum_{j=1}^{k} \frac{\theta_j - \theta_{j-1}}{\overline{\theta} - \underline{\theta}} . \frac{\theta_j + \theta_{j-1}}{2} \right) \right) = v' \left(\rho \left(\frac{\overline{\theta} + \underline{\theta}}{2} \right) \right)$$

$\Rightarrow \quad EU_{Gpartial} - EU_{Gno} > 0 \quad \Rightarrow \quad EU_{Gpartial} > EU_{Gno}$

SUMMARY: Relaxing assumption in section 3, consider a payoff function $v(.)$ which is strictly increasing and strictly concave, and satisfies Inada conditions. Furthermore, v is CRRA (Constant Relative Risk Aversion) or IRRA (Increasing Relative Risk Aversion). Then, for each state θ, the unique solution $\rho(\theta)$ of $v'(a) = \beta i \theta a$ satisfies that $v'(\rho(\theta)) = \beta i \theta \rho(\theta)$ is a strictly concave function of θ. Thereby, $EU_{Gfull} > EU_{Gpartial} > EU_{Gno}$, i.e. Proposition 3.2 holds.

4.2 Common prior belief $\mu_0(\theta)$

Instead of uniform prior distribution assumption, we continue by considering the case that $\mu_0(\theta)$ is a general cumulative distribution function. Similar to section 4.1, we obtains the following results:

Full information disclosure. Given state θ, there is a unique symmetric equilibrium at which everyone chooses action $\rho(\theta)$ satisfying:

$$v'(\rho(\theta)) = \beta i \theta \rho(\theta)$$

Partial information disclosure.

$$v'(\rho(s_j)) = \beta i \mathbb{E}_{\mu_0}[\theta|s_j]\rho(s_j)$$
$$\Rightarrow \rho(s_j) = \rho(\mathbb{E}_{\mu_0}[\theta|s_j]) \ \forall j = \overline{1,k}$$

No information disclosure.

$$v'(\rho(s)) = \beta i \mathbb{E}_{\mu_0}[\theta]\rho(s)$$
$$\Rightarrow \rho(s) = \rho(\mathbb{E}_{\mu_0}[\theta|s_j])$$

Therefore:

$$EU_{Gfull} = \int_{\underline{\theta}}^{\overline{\theta}} U_G(\rho(\theta),\theta)d\mu_0(\theta) = \int_{\underline{\theta}}^{\overline{\theta}} \left[v(\rho(\theta)) - \beta i\theta(\rho(\theta))^2\right] d\mu_0(\theta) - i\mathbb{E}_{\mu_0}[\theta]$$

$$EU_{Gpartial} = \sum_{j=1}^{k} v(\rho(\mathbb{E}_{\mu_0}[\theta|s_j])) \int_{\theta_{j-1}}^{\theta_j} d\mu_0(\theta) - \sum_{j=1}^{k} \beta i(\rho(\mathbb{E}_{\mu_0}[\theta|s_j]))^2 \int_{\theta_{j-1}}^{\theta_j} \theta d\mu_0(\theta) - i\mathbb{E}_{\mu_0}[\theta]$$

$$EU_{Gno} = v(\rho(\mathbb{E}_{\mu_0}[\theta])) - \beta i(\rho(\theta))^2\mathbb{E}_{\mu_0}[\theta] - i\mathbb{E}_{\mu_0}[\theta]$$

Hence:

(III) $EU_{Gpartial} - EU_{Gno}$

$$= \left[\mathbb{E}_{\mu_0}[\theta]\rho(\mathbb{E}_{\mu_0}[\theta]) - \sum_{j=1}^{k}\int_{\theta_{j-1}}^{\theta_j} d\mu_0(\theta)\mathbb{E}_{\mu_0}[\theta|s_j]\rho(\mathbb{E}_{\mu_0}[\theta|s_j])\right]\beta i\rho(\mathbb{E}_{\mu_0}[\theta])$$
$$= \left[v'(\rho(\mathbb{E}_{\mu_0}[\theta])) - \sum_{j=1}^{k}\int_{\theta_{j-1}}^{\theta_j} d\mu_0(\theta)v'(\rho(\mathbb{E}_{\mu_0}[\theta|s_j]))\right]\rho(\mathbb{E}_{\mu_0}[\theta])$$

Since $\sum_{j=1}^{k}\int_{\theta_{j-1}}^{\theta_j} d\mu_0(\theta) = \int_{\underline{\theta}}^{\overline{\theta}} d\mu_0(\theta) = 1$, if $v'(\rho(\theta)) = \beta i\theta\rho(\theta)$ is a strictly concave function of θ as achieved by **NOTE (∗)** in section 4.1, we obtain:

$$\sum_{j=1}^{k}\int_{\theta_{j-1}}^{\theta_j} d\mu_0(\theta)v'(\rho(\mathbb{E}_{\mu_0}[\theta|s_j])) < v'\left(\rho\left(\sum_{j=1}^{k}\int_{\theta_{j-1}}^{\theta_j} d\mu_0(\theta)\mathbb{E}_{\mu_0}[\theta|s_j]\right)\right) = v'(\rho(\mathbb{E}_{\mu_0}[\theta|s_j])$$

$$\Rightarrow \quad EU_{Gpartial} - EU_{Gno} > 0 \quad \Rightarrow \quad EU_{Gpartial} > EU_{Gno}$$

(IV) $EU_{Gfull} - EU_{Gpartial}$

$$= \sum_{j=1}^{k} \rho(\mathbb{E}_{\mu_0}[\theta|s_j]) \int_{\theta_{j-1}}^{\theta_j} d\mu_0(\theta)\left[v'\left(\rho(\mathbb{E}_{\mu_0}[\theta|s_j])\right) - \mathbb{E}_{\mu_0}[v'(\rho(\theta))|s_j]\right]$$

Here, if $v'(\rho(\theta)) = \beta i \theta \rho(\theta)$ is a strictly concave function of θ as achieved by **NOTE ($*$)** in section 4.1, we also have:

$$v'\left(\rho(\mathbb{E}_{\mu_0}[\theta|s_j])\right) > \mathbb{E}_{\mu_0}[v'(\rho(\theta))|s_j])$$
$$\Rightarrow \quad EU_{Gfull} - EU_{Gpartial} > 0 \quad \Rightarrow \quad EU_{Gfull} > EU_{Gpartial}$$

SUMMARY: Relaxing assumption in section 3, consider a general prior distribution $\mu_0(\theta)$. As developing in section 4.1, a payoff function $v(.)$ which is strictly increasing and strictly concave, and satisfies Inada conditions. Furthermore, v is CRRA (Constant Relative Risk Aversion) or IRRA (Increasing Relative Risk Aversion). Then, for each state θ, the unique solution $\rho(\theta)$ of $v'(a) = \beta i \theta a$ satisfies that $v'(\rho(\theta)) = \beta i \theta \rho(\theta)$ is a strictly concave function of θ. Thereby, $EU_{Gfull} > EU_{Gpartial} > EU_{Gno}$, i.e. Proposition 3.2 holds.

4.3 Structure of signalling rule $\sigma(s|\theta)$

In section 3, we only focus on signalling rule $\sigma(s|\theta)$ that has partition structure. This assumption is quite restrictive since Government has many ways to disclosure information about states, such as pooling very high state and very low state (Illustration for this Sender's strategy can be found in (Rayo and Segal, 2010)). For the sake of generalization, the following part will investigate whether full information disclosure is still optimal when all structures of signalling rule are taken into account.

Given a signalling rule, each signal realization s leads to a posterior belief $\mu(.|s) = \mu_s \in \Delta(\Theta)$. Accordingly, each signalling rule leads to a distribution over posterior belief. Define a distribution of posteriors by $\tau \in \Delta(\Delta(\Theta))$. Mathematically, a the signalling rule σ induces τ if $Supp(\tau) = \{\mu_s\}_{s \in S}$ and:

$$\mu_s(\theta) = \mu(\theta|s) = \frac{\sigma(s|\theta)\mu_0(\theta)}{\int\limits_{\Theta} \sigma(s|\theta')d\mu_0(\theta')} \quad \forall s, \theta$$
$$\tau(\mu) = \int\limits_{S} \mathbb{1}_{\mu_s = \mu} \int_{\Theta} \sigma(s|\theta)d\mu_0(\theta)ds \;\; \forall \mu$$

Then, belief μ (a conditional distribution) is induced by a signalling rule if τ is induced by that rule and $\tau(\mu) > 0$.

A distribution of posteriors τ is *Bayesian plausible* (Kamenica and Gentzkow, 2011) if the expected posterior probability equals the prior:

$$\int_{Supp(\tau)} \mu d\tau(\mu) = \mu_0$$

According to previous analysis, given a posterior belief μ_s, Public choose action $\rho(\mu_s)$ to solve the problem:

$$\max_{a \in A} \int_{\Theta} U_P(a, \theta) d\mu_s(\theta)$$

and there is an unique[3] symmetric Bayesian Nash equilibrium in which everyone chooses $\rho(\mu_s)$ satisfying:

$$v'(\rho(\mu_s)) = \beta i \mathbb{E}_{\mu_s}[\theta] \rho(\mu_s) \tag{3}$$

Hence, each distribution of posteriors τ determines a distribution of Receiver's actions. As from Sender's perspective, the Government's problem is then:

$$\max_{\tau} U_G(\rho(\mu), \theta) d\tau(\mu)$$

I now use a very similar approach of Mensch (2019) to establish my results.

Definition 1.

Firstly, consider two beliefs μ and μ', we assign them probabilities $\tau(\mu)$ and $\tau(\mu')$, respectively. Define an addition operation between μ and μ' such that the beliefs $\mu + \mu'$ is attached to the probability $\tau(\mu + \mu') = \tau(\mu) + \tau(\mu')$, where for any measurable $\Psi \in \Theta$:

$$(\mu + \mu')(\Psi) = \frac{\mu(\Psi)\tau(\mu) + \mu'(\Psi)\tau(\mu')}{\tau(\mu) + \tau(\mu')}$$

Subtraction operation is defined analogously. Furthermore, we allow scaling operation which is $\omega.\mu + \omega'.\mu'$ such that $\tau(\omega.\mu + \omega'.\mu') = \omega.\tau(\mu) + \omega'.\tau(\mu')$ and:

$$(\omega.\mu + \omega'.\mu')(\Psi) = \frac{\omega\mu(\Psi)\tau(\mu) + \omega'\mu'(\Psi)\tau(\mu')}{\omega.\tau(\mu) + \omega'.\tau(\mu')}$$

Definition 2.

Secondly, we define an ordering over posterior beliefs. A binary relation $\succeq_\sigma$ is *an ordering over beliefs* if the following properties are satisfied:

- *Completeness:* For any μ_1 and μ_2, either $\mu_1 \succeq_\sigma \mu_2$ or $\mu_2 \succeq_\sigma \mu_1$.
- *Transitivity:* For any μ_1, μ_2 and μ_3, if $\mu_1 \succeq_\sigma \mu_2$ and $\mu_2 \succeq_\sigma \mu_3$, then $\mu_1 \succeq_\sigma \mu_3$.

Obviously, there always exists an ordering over beliefs $\succeq_\sigma$ such that:

$$\mu_1 \succeq_\sigma \mu_2 \Leftrightarrow \rho(\mu_1) \geq \rho(\mu_2)$$

We say that this ordering is induced by the action chosen by the Receiver upon believing μ, where higher beliefs induce higher actions.

[3] The uniqueness of solution is guaranteed by assumptions of payoff function $v(.)$ in section 4.1.

Definition 3.

Define the marginal change in Sender's utility from adding two beliefs μ and μ' for which $\tau(\mu)$ and $\tau(\mu')$ are positive by $D(\mu,\mu')$. In the case of subtracting μ' from μ, this change will be $D(\mu,-\mu')$. Thus, when adding μ and $\epsilon.\mu'$:

$$D(\mu,\epsilon.\mu') = \frac{\tau(\mu)\left[\int U_G(\rho(\mu+\epsilon.\mu'),\theta)d\mu - \int U_G(\rho(\mu),\theta)d\mu\right] + \epsilon.\tau(\mu')\int U_G(\rho(\mu+\epsilon.\mu'),\theta)d\mu'}{\epsilon.\tau(\mu')}$$

Define $d(\mu,\epsilon.\mu')$ is the Gateaux derivative of the payoff of the sender from belief μ in the direction of μ':

$$d(\mu,\mu') = \lim_{\epsilon\to 0} D(\mu,\epsilon.\mu')$$

The following proposition[4] suggests a case in whih full information disclosure is optimal.

Proposition 3.

If the Sender's payoff are d-quasisubmodular, i.e. for $\theta' > \theta$ and $\mu' \succ_\sigma \mu$,

$$d(\mu',\theta) - d(\mu,\theta) \leq 0 \Rightarrow d(\mu',\theta') - d(\mu,\theta') < 0$$

Then, it is optimal for Sender to reveal all information.

Proof.

*Lemma 1. If the induced beliefs μ_1 first-order stochastically dominated μ_2 (denoted as $\mu_1 \succeq_{FOSD} \mu_2$), i.e. $\mu_1((-\infty,\theta]) \geq \mu_2((-\infty,\theta])$ for all $\theta \in \Theta$, then $\mu_2 \succeq_\sigma \mu_1$. Indeed, note that $\mathbb{E}_\mu[\theta] = \int \theta d(\theta)$. Since $f(\theta) = \theta$ is an increasing function, it follows that if $\mu_1 \succeq_{FOSD} \mu_2$, then $\mathbb{E}_{\mu_1}[\theta] \geq \mathbb{E}_{\mu_2}[\theta]$. From equation (3), we obtain $\rho(\mu_1) \leq \rho(\mu_2)$. Hence, $\mu_2 \succeq_\sigma \mu_1$.

*The intuition for proof of proposition 3 as follows: Suppose that full information disclosure is not optimal for Sender. Then with the optimal signalling rule, there exists a posterior μ with more than one state in the support. We split μ into two equiprobable posteriors μ_1 and μ_2 and perturb them so that a little more weight is placed at the bottom (or a little less weight is placed at the top) of the support of μ_2, while the opposite is done with μ_1. As a result, $\mu_1 \succeq_{FOSD} \mu_2$, thereby $\mu_2 \succeq_\sigma \mu_1$. Therefore, it will be strict improvement to swap even more of the support. Since the perturbation is arbitrarily small, there will be a strict improvement over this initial posterior (A contradiction).

[4]Based on Theorem 4, Mensch (2019). However, this paper considers the case in which high state induces high action of Receiver, opposite to the problem we are discussing.

*Algebraically, suppose that the signalling rule $\sigma(s|\theta)$ is optimal for Government's problem, where there exists some signal s such that the posterior $\mu_s(\theta) = \mu(\theta|s) \notin \{0, 1\}$ for some θ. Then we consider an equivalent optimal signalling rule $\sigma'(s|\theta)$ which is identical to $\sigma(s|\theta)$ except that signal s is duplicated into s_1 and s_2 such that two posteriors μ_{s_1} and μ_{s_2} satisfy: $\mu_{s_1} = \mu_{s_2} = \mu_s$ and $\tau(\mu_{s_1}) = \tau(\mu_{s_2}) = \frac{1}{2}\tau(\mu_s)$. For conciseness, we denote $\mu_1 = \mu_{s_1}$ and $\mu_2 = \mu_{s_2}$.
Let $\underline{\theta} \equiv \min\{\theta \in supp(\mu(\theta|s))\}$ and $\overline{\theta} \equiv \max\{\theta \in supp(\mu(\theta|s))\}$. Let a constant m satisfy: $0 < m < \min\{\mu(\underline{\theta}|s), \mu(\overline{\theta}|s)\}$.
Then for any $\alpha \in (0, m)$,

$$\mu_1 + \alpha\overline{\theta} - \alpha\underline{\theta} \succ_{FOSD} \mu_2 - \alpha\overline{\theta} + \alpha\underline{\theta}$$
$$\Rightarrow \mu_2 - \alpha\overline{\theta} + \alpha\underline{\theta} \succ_{\sigma} \mu_1 + \alpha\overline{\theta} - \alpha\underline{\theta}$$

Since $\overline{\theta} > \underline{\theta}$, by d-quasisubmodularity, we have either

$$d(\mu_1 + \alpha\overline{\theta} - \alpha\underline{\theta}, \underline{\theta}) - d(\mu_2 - \alpha\overline{\theta} + \alpha\underline{\theta}, \underline{\theta}) < 0 \tag{4}$$

or

$$d(\mu_2 - \alpha\overline{\theta} + \alpha\underline{\theta}, \overline{\theta}) - d(\mu_1 + \alpha\overline{\theta} - \alpha\underline{\theta}, \overline{\theta}) < 0 \tag{5}$$

Moreover,

$$\begin{aligned}
(4) &\Rightarrow d(\mu_1 + \alpha\overline{\theta} - \alpha\underline{\theta}, (m-\alpha)\underline{\theta}) - d(\mu_2 - \alpha\overline{\theta} + \alpha\underline{\theta}, (m-\alpha)\underline{\theta}) < 0 \\
&\Rightarrow 0 < d(\mu_2 - \alpha\overline{\theta} + \alpha\underline{\theta}, (m-\alpha)\underline{\theta}) + d(\mu_1 + \alpha\overline{\theta} - \alpha\underline{\theta}, -(m-\alpha)\underline{\theta}) \\
&\Rightarrow \tau(\mu_1 + \alpha\overline{\theta} - \alpha\underline{\theta}) \int U_G(\rho(\mu_1 + \alpha\overline{\theta} - \alpha\underline{\theta}), \theta) d(\mu_1 + \alpha\overline{\theta} - \alpha\underline{\theta})(\theta) \\
&\quad + \tau(\mu_2 - \alpha\overline{\theta} + \alpha\underline{\theta}) \int U_G(\rho(\mu_2 - \alpha\overline{\theta} + \alpha\underline{\theta}), \theta) d(\mu_2 - \alpha\overline{\theta} + \alpha\underline{\theta})(\theta) \\
&\quad < \tau(\mu_1 + \alpha\overline{\theta} - m\underline{\theta}) \int U_G(\rho(\mu_1 + \alpha\overline{\theta} - m\underline{\theta}), \theta) d(\mu_1 + \alpha\overline{\theta} - m\underline{\theta})(\theta) \\
&\quad + \tau(\mu_2 - \alpha\overline{\theta} + m\underline{\theta}) \int U_G(\rho(\mu_2 - \alpha\overline{\theta} + m\underline{\theta}), \theta) d(\mu_2 - \alpha\overline{\theta} + m\underline{\theta})(\theta)
\end{aligned}$$

$$\begin{aligned}
(5) &\Rightarrow d(\mu_2 - \alpha\overline{\theta} + \alpha\underline{\theta}, (m-\alpha)\overline{\theta}) - d(\mu_1 + \alpha\overline{\theta} - \alpha\underline{\theta}, (m-\alpha)\overline{\theta}) < 0 \\
&\Rightarrow 0 < d(\mu_1 + \alpha\overline{\theta} - \alpha\underline{\theta}, (m-\alpha)\overline{\theta}) + d(\mu_2 - \alpha\overline{\theta} + \alpha\underline{\theta}, -(m-\alpha)\overline{\theta}) \\
&\Rightarrow \tau(\mu_1 + \alpha\overline{\theta} - \alpha\underline{\theta}) \int U_G(\rho(\mu_1 + \alpha\overline{\theta} - \alpha\underline{\theta})) d(\mu_1 + \alpha\overline{\theta} - \alpha\underline{\theta})(\theta) \\
&\quad + \tau(\mu_2 - \alpha\overline{\theta} + \alpha\underline{\theta}) \int U_G(\rho(\mu_2 - \alpha\overline{\theta} + \alpha\underline{\theta})) d(\mu_2 - \alpha\overline{\theta} + \alpha\underline{\theta})(\theta) \\
&\quad < \tau(\mu_1 + m\overline{\theta} - \alpha\underline{\theta}) \int U_G(\rho(\mu_1 + m\overline{\theta} - \alpha\underline{\theta})) d(\mu_1 + m\overline{\theta} - \alpha\underline{\theta})(\theta) \\
&\quad + \tau(\mu_2 - m\overline{\theta} + \alpha\underline{\theta}) \int U_G(\rho(\mu_2 - m\overline{\theta} + \alpha\underline{\theta})) d(\mu_2 - m\overline{\theta} + \alpha\underline{\theta})(\theta)
\end{aligned}$$

Consider the sequence $\{\alpha_k\}_{k\in\mathbb{N}^*}$ defined by: $\alpha_k = \frac{m}{2^k}$. Then $\alpha_k \in (0, m)$ $\forall k \in \mathbb{N}^*$ and $\lim_{k\to\infty} \alpha_k = 0$. Hence, above inequalities holds for all α_k ($k \in \mathbb{N}^*$). In the limit $k \to \infty$:

$$\tau(\mu_1)\int U_G(\rho(\mu_1))d\mu_1(\theta) + \tau(\mu_2)\int U_G(\rho(\mu_2))d\mu_2(\theta)$$
$$\leq \tau(\mu_1 - m\underline{\theta})\int U_G(\rho(\mu_1 - m\underline{\theta}))d(\mu_1 - m\underline{\theta})(\theta) + \tau(\mu_2 + m\underline{\theta})\int U_G(\rho(\mu_1 + m\underline{\theta}))d(\mu_2 + m\underline{\theta})(\theta)$$

or

$$\tau(\mu_1)\int U_G(\rho(\mu_1))d\mu_1(\theta) + \tau(\mu_2)\int U_G(\rho(\mu_2))d\mu_2(\theta)$$
$$\leq \tau(\mu_1 + m\overline{\theta})\int U_G(\rho(\mu_1 + m\overline{\theta}))d(\mu_1 + m\overline{\theta})(\theta) + \tau(\mu_2 - m\overline{\theta})\int U_G(\rho(\mu_2 - m\overline{\theta}))d(\mu_2 - m\overline{\theta})(\theta)$$

Without loss of generality, suppose that the former is true. Additionally, $\mu_2 + m\underline{\theta} \succ_\sigma \mu_1 - m\underline{\theta}$. Then there exists further improvement by either adding small enough ϵ of $\underline{\theta}$ from $\mu_1 - m\underline{\theta}$ to $\mu_2 + m\underline{\theta}$ or adding small enough ϵ of $\overline{\theta}$ from $\mu_2 + m\underline{\theta}$ to $\mu_1 - m\underline{\theta}$, due to the similar logic to inequalities (4) and (5), respectively. This improvement is strict, thus the signalling rule $\sigma'(s|\theta)$ (as well as $\sigma(s|\theta)$) is not optimal (a contradiction). Therefore, full information disclosure is optimal. (Q.E.D)

Check for d-quasisubmodular

Firstly, we sequentially examine Individual's problem and respective equilibrium action of Public when posteriors are μ and $\mu + \epsilon\mu'$.

Given posterior μ, individual maximizes:

$$\int U_p(a,\theta)d\mu$$

F.O.C

$$\int \left[v'(\rho(\mu)) + \beta i z\theta\right] d\mu = 0$$

In symmetric equilibrium:

$$\int \left[v'(\rho(\mu)) + \beta i \rho(\mu)\theta\right] d\mu = 0 \tag{6}$$

Given posterior $\mu + \epsilon\mu'$, individual maximizes:

$$\int U_p(a,\theta)d(\mu + \epsilon\mu') = \frac{\tau(\mu)}{\tau(\mu) + \epsilon\tau(\mu')}\int U_P(\rho(\mu + \epsilon\mu'),\theta)d\mu + \frac{\epsilon\tau(\mu')}{\tau(\mu) + \epsilon\tau(\mu')}\int U_P(\rho(\mu + \epsilon\mu'),\theta)d\mu'$$

F.O.C

$$\frac{\tau(\mu)}{\tau(\mu)+\epsilon\tau(\mu')}\int\left[v'((\rho(\mu+\epsilon\mu'))-\beta iz\theta\right]d\mu+\frac{\epsilon\tau(\mu')}{\tau(\mu)+\epsilon\tau(\mu')}\int\left[v'((\rho(\mu+\epsilon\mu'))-\beta iz\theta\right]d\mu'=0$$

In symmetric equilibrium:

$$\frac{\tau(\mu)}{\tau(\mu)+\epsilon\tau(\mu')}\int\left[v'((\rho(\mu+\epsilon\mu'))-\beta i\rho(\mu+\epsilon\mu')\theta\right]d\mu \\ +\frac{\epsilon\tau(\mu')}{\tau(\mu)+\epsilon\tau(\mu')}\int\left[v'((\rho(\mu+\epsilon\mu'))-\beta i\rho(\mu+\epsilon\mu')\theta\right]d\mu'=0 \quad (7)$$

Subtracting the equations (7) and (6), we obtain:

$$\frac{\tau(\mu)}{\tau(\mu)+\epsilon\tau(\mu')}\int\left\{\left[v'(\rho(\mu+\epsilon\mu')-v'(\rho(\mu)\right]-\beta i\theta\left[\rho(\mu+\epsilon\mu')-\rho(\mu)\right]\right\}d\mu \\ +\frac{\epsilon\tau(\mu')}{\tau(\mu)+\epsilon\tau(\mu')}\int\left[v'(\rho(\mu+\epsilon\mu'))-\beta i\theta\rho(\mu+\epsilon\mu')\right]d\mu'=0$$

Using Taylor expansion,

$$\frac{\tau(\mu)}{\tau(\mu)+\epsilon\tau(\mu')}\int\left\{v''(\rho(\mu))\left[\rho(\mu+\epsilon\mu')-\rho(\mu)\right]-\beta i\theta\left[\rho(\mu+\epsilon\mu')-\rho(\mu)\right]\right\}d\mu \\ +\frac{\epsilon\tau(\mu')}{\tau(\mu)+\epsilon\tau(\mu')}\int\left[v'(\rho(\mu+\epsilon\mu')-\beta i\theta\rho(\mu+\epsilon\mu')\right]d\mu'=0$$

Hence,

$$\rho(\mu+\epsilon\mu')-\rho(\mu)=-\frac{\epsilon\tau(\mu')}{\tau(\mu)}.\frac{\int\left[v'(\rho(\mu+\epsilon\mu')-\beta i\theta\rho(\mu+\epsilon\mu')\right]d\mu'}{\int\left[v''(\rho(\mu))-\beta i\theta\right]}d\mu$$

With this change in equilibrium action level, we can calculate the marginal change in Government's utility from adding the amount $\epsilon.\mu'$ to the belief μ:

$$\begin{aligned}D(\mu,\epsilon.\mu')&=\frac{\tau(\mu)\left[\int U_G(\rho(\mu+\epsilon.\mu'),\theta)d\mu-\int U_G(\rho(\mu),\theta)d\mu\right]+\epsilon.\tau(\mu')\int U_G(\rho(\mu+\epsilon.\mu'),\theta)d\mu'}{\epsilon.\tau(\mu')}\\&=\frac{\tau(\mu)}{\epsilon.\tau(\mu')}\int\frac{\partial U_G}{\partial\rho}(\rho(\mu+\epsilon.\mu'),\theta)d\mu\left[\rho(\mu+\epsilon.\mu')-\rho(\mu)\right]+\int U_G(\rho(\mu+\epsilon.\mu'),\theta)d\mu'\\&=-\frac{\int\left[v'(\rho(\mu))-2\beta i\theta\rho(\mu)\right]d\mu\int\left[v'(\rho(\mu+\epsilon\mu')-\beta i\theta\rho(\mu+\epsilon\mu')\right]d\mu'}{\int\left[v''(\rho(\mu))-\beta i\theta\right]d\mu}\\&\quad+\int\left[v(\rho(\mu+\epsilon.\mu'))-\beta i\theta(\rho(\mu+\epsilon.\mu'))^2-i\theta\right]d\mu'\end{aligned}$$

The marginal change in Government's utility from adding the amount $\epsilon.\theta'$ to the belief μ then is:

$$D(\mu, \epsilon.\theta') = -\frac{\int [v'(\rho(\mu)) - 2\beta i\theta\rho(\mu)]\, d\mu\, [v'(\rho(\mu + \epsilon\theta') - \beta i\theta'\rho(\mu + \epsilon\theta')]}{\int [v''(\rho(\mu)) - \beta i\theta]\, d\mu} + \Big[v(\rho(\mu + \epsilon.\theta')) - \beta i\theta'(\rho(\mu + \epsilon.\theta'))^2 - i\theta'\Big]$$

Hence,

$$\begin{aligned}
d(\mu, \theta') &= \lim_{\epsilon\to 0} D(\mu, \epsilon.\theta') \\
&= \Big[v(\rho(\mu)) - \beta i\theta'(\rho(\mu))^2 - i\theta'\Big] - \frac{[v'(\rho(\mu)) - \beta i\theta'\rho(\mu)] \int [v'(\rho(\mu)) - 2\beta i\theta\rho(\mu)]\, d\mu}{\int [v''(\rho(\mu)) - \beta i\theta]\, d\mu} \\
&= \Big[v(\rho(\mu)) - \beta i\theta'(\rho(\mu))^2 - i\theta'\Big] - \frac{[v'(\rho(\mu)) - \beta i\theta'\rho(\mu)]\, [v'(\rho(\mu)) - 2\beta i\mathbb{E}_\mu[\theta]\rho(\mu)]}{v''(\rho(\mu)) - \beta i\mathbb{E}_\mu[\theta]} \\
&= \Big[v(\rho(\mu)) - \beta i\theta'(\rho(\mu))^2 - i\theta'\Big] + \frac{[v'(\rho(\mu)) - \beta i\theta'\rho(\mu)]\, v'(\rho(\mu))}{v''(\rho(\mu)) - \frac{v'(\rho(\mu)}{\rho(\mu)}}
\end{aligned}$$

An economic interpretation for $d(\mu, \theta')$ as follows. The first term of the expression is the marginal utility of Government from having action $\rho(\mu)$ taken when the state is θ'. The second term is the marginal effect on Government's utility from Public changing their action from that taken at posterior μ since more weight is placed on θ'.
To check for d-submodularity, we separate the sum of the terms involving both action $\rho(\mu)$ and state θ' which is:

$$\begin{aligned}
f(\rho(\mu), \theta') &= -\beta i\theta'(\rho(\mu))^2 - \frac{\beta i\theta'\rho(\mu)v'(\rho(\mu))}{v''(\rho(\mu)) - \frac{v'(\rho(\mu)}{\rho(\mu)}} \\
&= -\frac{\beta i\theta'(\rho(\mu))^2 v''(\rho(\mu))}{v''(\rho(\mu)) - \frac{v'(\rho(\mu)}{\rho(\mu)}} \\
\Rightarrow \qquad \frac{\partial^2 f}{\partial\theta'\partial\rho(\mu)} &= -\frac{d}{d\rho(\mu)}\left[\frac{\beta i(\rho(\mu))^2 v''(\rho(\mu))}{v''(\rho(\mu)) - \frac{v'(\rho(\mu)}{\rho(\mu)}}\right]
\end{aligned}$$

NOTE$(**)$
Since $v' > 0, v'' < 0$ and $\rho > 0$, we obtain that the condition for Government's utility is d-submodular is equivalent to:

$$\frac{\partial^2 f}{\partial\theta'\partial\rho(\mu)} < 0 \Leftrightarrow \frac{d}{d\rho}\left(\frac{\beta i\rho^2 v''}{v'' - \frac{v'}{\rho}}\right) > 0 \Leftrightarrow \frac{d}{d\rho}\left(\frac{\rho^3 v''}{\rho v'' - v'}\right) > 0 \Leftrightarrow \frac{d}{d\rho}\left[\left(\frac{v'}{\rho}\right)^2 \div \frac{-\rho v''}{v'}\right] < 0 \quad (8)$$

Interestingly, condition (8) is very closed to (2) in **NOTE**$(*)$. Since $\frac{v'}{\rho}$ is always increasing in ρ given the assumption that $v'' < 0$, we can provide the same sufficient conditions for payoff function v such that condition (8) is satisfied. That is the further assumption that payoff function v is CRRA (Constant Relative Risk Aversion) or IRRA (Increasing Relative Risk Aversion). Then, Government's utility is d-submodular, thereby being d-quasisubmodular. From Proposition 3, full information disclosure is optimal. This result is consistent with findings in the canonical model (section 3) and the robustness in two above parts (4.1 and 4.2).

5 Discussion and concluding remarks

The paper aims to formulate a model of Government information disclosure to the Public during an epidemic, using the framework of Information Design.

Firstly, considering full information about the severity of disease (state of the world) as a benchmark, there exists an incentive misalignment between the Government/ benevolent social planner and the Public. This differential stems from the logic as follows: An increase in each individual's social activity level raises the expected cost of social activity for others by increasing their probability of getting infected per unit social activity. On the one hand, a self-interested individual does not take into account his/her negative influence on other people's well-being when making his/her decision. On the other hand, the Government – with the objective of maximizing social welfare – takes these external effects into consideration. The social marginal cost of public activity incorporates all the harmful external effects of the whole population, thereby being higher than the private marginal cost of public activity. Consequently, individuals are more likely to choose an excessive level of social activity in equilibrium, while social welfare would be improved if their social activity level is reduced slightly.

Secondly, deviating from the benchmark, we empower the Government by formalizing the notation of information advantage. It is more realistic since, in an epidemic, especially the early stage, the Government is better informed about the disease severity (death counts, the capacity of health system, biohazard level of virus, etc.) in comparison to the Public. If such is the case, is there any opportunity for the Government to manipulate the Public's beliefs to increase social welfare? The answer depends upon the pre-commitment power of the Government.

When the Government (Sender) lacks commitment power (Cheap talk - Crawford and Sobel (1982)), the Government is only able to communicate to maximize the interim social welfare. Under asymmetric information, the problem simply becomes a game in which

Sender's choice and Receiver's choice are strategically 'simultaneous', Communication failure emerges in the equilibrium and leads to public underreactions when the state is more severe and public overreactions when the state is less severe compared to the prior mean. Intuitively speaking, even if the Government declares truthfully about the epidemic situation, rational individuals would believe that the Government just overstates the problem to encourage lower social activity which benefits social welfare. Consequently, the Public underestimate messages from the Government, which leads the Government to exaggerate even more. In equilibrium, the Government always delivers the same announcements regardless of the true severity. The specific content of Government's messages becomes meaningless and any communication forms turn to cheap talks (Yue and Yixi, 2020).

When The Government (Sender) possesses commitment power (Bayesian persuasion - Kamenica and Gentzkow (2011)), the Government could choose how to disclose information to maximize its ex-ante expected utility. Commitment power is an essential notation in Bayesian Persuasion since it allows the Sender to eliminate needless worries about the Receiver's interpretation of his actions, and the Receiver simply solves decision problem given the information provided by the Sender. Compared to the Cheap Talk model, commitment power rules out the possibility of babbling equilibrium and related coordination failures. With respects to the Government disclosure problem, the Government could exploit its commitment power to choose optimal signalling rule. The canonical model in this paper theoretically predicts that any partial information disclosure with partition structure is worse than full information disclosure and better than no information disclosure, in terms of ex-ante social welfare. As discussed in extensions, I specified the sufficient condition for the optimality of full information disclosure in the general setup with any prior belief and any signalling rule structure: each individual has the payoff function of social activity level which is increasing, concave, and CRRA (Constant Relative Risk Aversion) or IRRA (Increasing Relative Risk Aversion). For some DRRA (Decreasing Relative Risk Aversion) payoff function, this result may not hold.

Although necessary condition for the optimality of information disclosure as well as examining when partial disclosure could be optimal need further researched, this paper still suggests some policy implications on how information should be conveyed to the public. In the context of coronavirus disease (COVID-19) outbreak, many countries worldwide implement extraordinary measures such as restriction of mass gathering, social distancing and self-quarantine to alleviate negative impacts on the well-being of the whole population. Notwithstanding available enforcement, the efficiency of these methods critically depends on individual voluntary compliance, which is significantly different around the world (Paola and Imran, 2020). Commitment power of the Government could be one important element which helps explain the effectiveness of such practices. In some countries with low trust in the institution, communication failure arises as a consequence of lacking truth-telling commitment from the Government. In contrast, while COVID-19 is an unprecedented shock which hinders the authority from establishing reputation, there still exist countries who can build some forms of commitment power through collaborating with stakeholders (for examples, delegating prominent public health experts to make public announcements) or utilizing effects of culture, social capital and partisan. Such efforts would enable the Government to employ persuasion strategies as an Information Designer, thereby achieving better ex-ante social welfare and avoiding public underreaction or overreaction. This paper argues that as Government is endowed commitment capability, full information disclosure (transparency, keeping promises with the Public) instead of pooling some levels of disease severity would be optimal.

The generalisability of these findings in this paper is subject to certain limitations. Firstly, heterogeneous individuals problem has not been taken into account in the paper, though persuasion of privately informed receivers is also proposed in the Information Design literature. In fact, during an epidemic like the COVID-19 outbreak, individuals could have very different likelihood of being infected, for instance, the elder is more vulnerable than the younger (Miltiadis, 2020). Additionally, along with the prevalence of media, individuals may encounter multiple stories, incessant headlines and continuous updates, thus they may have different private channels of information about the disease severity, which plays a role as their own prior information when the Government make some official announcements in each period. This issue should be integrating into the extended model. Secondly, the presumptions regarding Bayesian rationality of Receivers and no cost of messages of Sender are quite strict. Some results in this study may no longer hold when we consider alternative assumptions, for example, as some suggestions from Mensch (2019)'s paper, with the presence of Sender's cost function of messages, full information disclosure could be not optimal. Even if this is the case, the structure of the optimal partial information disclosure still needs to be researched in more details. I hope this topic could be addressed in future studies./

References

H. Allcott, L. Boxell, J. Conway, M. Gentzkow, M. Thaler, and D. Y. Yang. Polarization and public health: Partisan differences in social distancing during the coronavirus pandemic. *NBER Working Paper*, (w26946), 2020.

E. Azarmsa and L. W. Cong. Persuasion in relationship finance. *Journal of Financial Economics*, 2020.

D. Bergemann and S. Morris. Robust predictions in games with incomplete information. *Econometrica*, 81(4):1251–1308, 2013.

D. Bergemann and S. Morris. Bayes correlated equilibrium and the comparison of information structures in games. *Theoretical Economics*, 11(2):487–522, 2016a.

D. Bergemann and S. Morris. Information design, bayesian persuasion, and bayes correlated equilibrium. *American Economic Review*, 106(5):586–91, 2016b.

D. Bergemann and S. Morris. Information design: A unified perspective. *Journal of Economic Literature*, 57(1):44–95, 2019.

R. Boleslavsky and C. Cotton. Grading standards and education quality. *American Economic Journal: Microeconomics*, 7(2):248–79, 2015.

L. Bursztyn, A. Rao, C. Roth, and D. Yanagizawa-Drott. Misinformation during a pandemic. *University of Chicago, Becker Friedman Institute for Economics Working Paper*, (2020-44), 2020.

F. Chen. A mathematical analysis of public avoidance behavior during epidemics using game theory. *Journal of theoretical biology*, 302:18–28, 2012.

V. P. Crawford and J. Sobel. Strategic information transmission. *Econometrica: Journal of the Econometric Society*, pages 1431–1451, 1982.

S. Das, E. Kamenica, and R. Mirka. Reducing congestion through information design. In *2017 55th Annual Allerton Conference on Communication, Control, and Computing (Allerton)*, pages 1279–1284. IEEE, 2017.

P. Dworczak and G. Martini. The simple economics of optimal persuasion. *Journal of Political Economy*, 127(5):1993–2048, 2019.

M. Farboodi, G. Jarosch, and R. Shimer. Internal and external effects of social distancing in a pandemic. Technical report, National Bureau of Economic Research, 2020.

M. Gentzkow and E. Kamenica. A rothschild-stiglitz approach to bayesian persuasion. *American Economic Review*, 106(5):597–601, 2016.

P.-Y. Geoffard and T. Philipson. Rational epidemics and their public control. *International economic review*, pages 603–624, 1996.

Y. Guo and E. Shmaya. The interval structure of optimal disclosure. *Econometrica*, 87(2): 653–675, 2019.

M. Ivanov. Optimal signals in bayesian persuasion mechanisms with ranking. *Work. Pap., McMaster Univ., Hamilton, Can*, 2015.

E. Kamenica. Bayesian persuasion and information design. *Annual Review of Economics*, 11:249–272, 2019.

E. Kamenica and M. Gentzkow. Bayesian persuasion. *American Economic Review*, 101(6): 2590–2615, 2011.

W. O. Kermack and A. G. McKendrick. A contribution to the mathematical theory of epidemics. *Proceedings of the royal society of london. Series A, Containing papers of a mathematical and physical character*, 115(772):700–721, 1927.

A. Kolotilin. Optimal information disclosure: A linear programming approach. *Theoretical Economics*, 13(2):607–635, 2018.

A. Kolotilin and A. Wolitzky. Assortative information disclosure. 2020.

A. Kolotilin, T. Mylovanov, A. Zapechelnyuk, and M. Li. Persuasion of a privately informed receiver. *Econometrica*, 85(6):1949–1964, 2017.

S. Kushner Gadarian, S. W. Goodman, and T. B. Pepinsky. Partisanship, health behavior, and policy attitudes in the early stages of the covid-19 pandemic. *Health Behavior, and Policy Attitudes in the Early Stages of the COVID-19 Pandemic (March 27, 2020)*, 2020.

H. Li and X. Shi. Discriminatory information disclosure. *American Economic Review*, 107 (11):3363–85, 2017.

L. Mariani, J. Gagete-Miranda, and P. Retti. Words can hurt: How political communication can change the pace of an epidemic. *DOI: https://doi. org/10.31235/osf. io/a32r7*, 2020.

L. Mathevet, J. Perego, and I. Taneva. On information design in games. *Journal of Political Economy*, 128(4):1370–1404, 2020.

L. Matyskova. Bayesian persuasion with costly information acquisition. *CERGE-EI Working Paper Series*, (614), 2018.

J. Mensch. Monotone persuasion. *Available at SSRN 3265980*, 2019.

P. R. Milgrom. Good news and bad news: Representation theorems and applications. *The Bell Journal of Economics*, pages 380–391, 1981.

M. Miltiadis. Covid and social distancing with a heterogenous population. Technical report, University of Kent - School of Economics Discussion Papers, 2020.

C. Minaudier. The value of confidential policy information: persuasion, transparency, and influence. *Unpublished manuscript*, 2018.

M. Ostrovsky and M. Schwarz. Information disclosure and unraveling in matching markets. *American Economic Journal: Microeconomics*, 2(2):34–63, 2010.

M. Painter and T. Qiu. Political beliefs affect compliance with covid-19 social distancing orders. *Available at SSRN 3569098*, 2020.

G. Paola and R. Imran. Compliance with social distancing during the covid-19 crisis. Technical report, CEPR, 2020.

C. Rafkin, A. Shreekumar, and P.-L. Vautrey. When guidance changes: Government inconsistency and public beliefs. 2020.

L. Rayo and I. Segal. Optimal information disclosure. *Journal of political Economy*, 118(5): 949–987, 2010.

M. Spense. Job market signaling. *The Quarterly Journal of Economics*, 87(3):355–374, 1973.

I. Taneva. Information design. *American Economic Journal: Microeconomics*, 11(4):151–85, 2019.

L. Yue and Y. Yixi. Information asymmetry in an epidemic: An equilibrium analysis of communication failure. 2020.